SIGNALS & SYSTEMS

Complete Multipage Master Handbook & Problem Solver | GATE ECE 2027

Target Exam:	GATE 2027 Electronics & Communication (EC)
Organizing Institute:	IIT Madras
Syllabus Standard:	100% Complete Syllabus + 2027 Revisions
Edition:	Comprehensive Multipage Textbook & Problem Solver

TABLE OF CONTENTS

CHAPTER 1: CONTINUOUS & DISCRETE SIGNAL FUNDAMENTALS

1.1 Signal Classification & Energy/Power

Signals are classified based on energy, power, periodicity, and symmetry.

- **Energy Signal:** 0 < E < infinity and Power P = 0. Energy E = integral |x(t)|^2 dt.
- **Power Signal:** 0 < P < infinity and Energy E = infinity.
- **Even Signal:** x(-t) = x(t) (Real & even Fourier transform).
- **Odd Signal:** x(-t) = -x(t) (Imaginary & odd Fourier transform).

CHAPTER 2: LTI SYSTEMS & CONVOLUTION

2.1 Continuous & Discrete Convolution

LTI systems are completely characterized by their impulse response h(t).

- **Continuous Convolution:** y(t) = integral x(tau) h(t - tau) dtau.
- **Discrete Convolution:** y(n) = sum x(k) h(n - k).
- **Length Rule:** If x(n) has length L and h(n) has length M, output y(n) has length $N_y = L + M - 1$.

CHAPTER 3: FOURIER TRANSFORMS & LAPLACE/Z TRANSFORMS

3.1 Transform Properties & ROC Rules

Transforms convert time-domain differential/difference equations into algebraic equations.

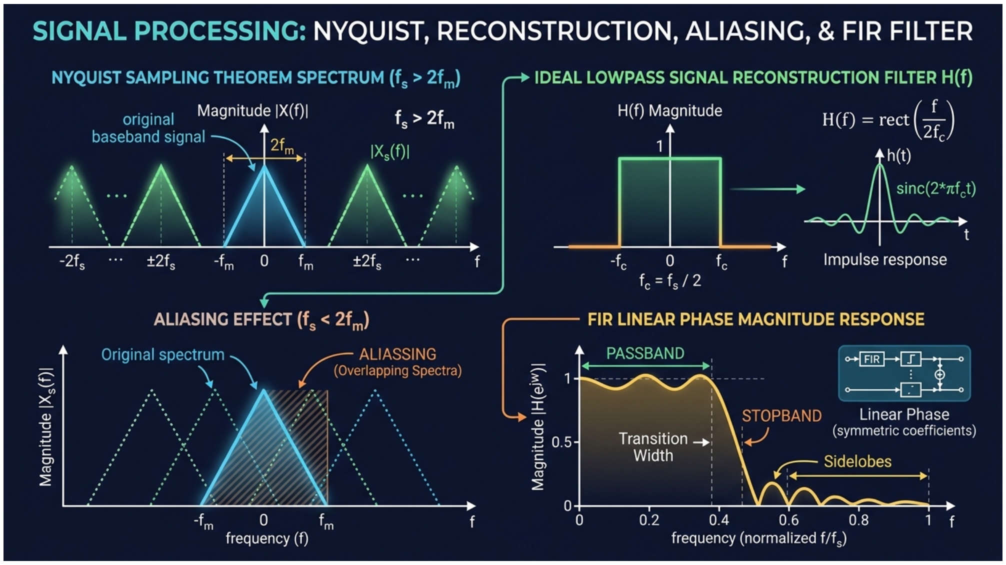
- **Laplace ROC:** Contains no poles. Stable if ROC contains j*omega axis.
- **Z-Transform ROC:** Stable if ROC contains unit circle $|z| = 1$.

CHAPTER 4: NYQUIST SAMPLING & SIGNAL RECONSTRUCTION

4.1 Nyquist Theorem & Ideal Reconstruction Filter (GATE 2027 Focus)

A continuous signal x(t) band-limited to f_m Hz can be perfectly reconstructed if sampled at rate f_s >= 2 f_m.

- **Nyquist Rate:** Minimum sampling rate f_s = 2 f_m.
- **Aliasing:** Occurs when f_s < 2 f_m, causing spectral overlap.
- **Reconstruction Formula:** x(t) = sum x(n T_s) sinc((t - n T_s)/T_s).



CHAPTER 5: FIR AND IIR DIGITAL FILTER DESIGN

5.1 FIR & IIR Filter Design Methods (NEW FOR GATE 2027)

Digital filter design is a major newly added topic in the GATE 2027 syllabus.

- **FIR Filters:** Always stable, exact linear phase when $h(n) = +/- h(N - 1 - n)$. Designed using Window methods (Rectangular, Hamming, Blackman).
- **IIR Filters & Bilinear Transformation (BLT):** Maps s-plane to z-plane using $s = (2/T_s) * (1 - z^{-1})/(1 + z^{-1})$.
- **Frequency Pre-Warping:** $\Omega = (2/T_s) * \tan(\omega/2)$ to compensate for non-linear frequency compression.

WORKED GATE NUMERICAL: Bilinear Transformation Filter Pole
Problem: Convert analog filter $H_a(s) = 1 / (s + 5)$ to digital filter $H(z)$ using BLT with $T_s = 0.2$ s.
Step-by-Step Derivation & Solution: 1. Substitute $s = 10 * (z - 1)/(z + 1)$ 2. $H(z) = 1 / [10(z-1)/(z+1) + 5] = (z+1) / (15z - 5) = (1/15) * (1 + z^{-1}) / (1 - (1/3)z^{-1})$ 3. Digital Pole: **z = 1/3 = 0.333** (Inside unit circle -> Stable).